

# One-Dimensional Wave Bottom Boundary Layer Model Comparison: Specific Eddy Viscosity and Turbulence Closure Models

Jack A. Puleo<sup>1</sup>; Oleg Mouraenko<sup>2</sup>; and Daniel M. Hanes<sup>3</sup>

**Abstract:** Six one-dimensional-vertical wave bottom boundary layer models are analyzed based on different methods for estimating the turbulent eddy viscosity: Laminar, linear, parabolic,  $k$ —one equation turbulence closure,  $k-\varepsilon$ —two equation turbulence closure, and  $k-\omega$ —two equation turbulence closure. Resultant velocity profiles, bed shear stresses, and turbulent kinetic energy are compared to laboratory data of oscillatory flow over smooth and rough beds. Bed shear stress estimates for the smooth bed case were most closely predicted by the  $k-\omega$  model. Normalized errors between model predictions and measurements of velocity profiles over the entire computational domain collected at 15° intervals for one-half a wave cycle show that overall the linear model was most accurate. The least accurate were the laminar and  $k-\varepsilon$  models. Normalized errors between model predictions and turbulence kinetic energy profiles showed that the  $k-\omega$  model was most accurate. Based on these findings, when the smallest overall velocity profile prediction error is required, the processing requirements and error analysis suggest that the linear eddy viscosity model is adequate. However, if accurate estimates of bed shear stress and TKE are required then, of the models tested, the  $k-\omega$  model should be used.

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## Introduction

Nearshore surface wave motions induce flow oscillations near the sea bed that are altered by frictional resistance in what is termed the wave bottom boundary layer (WBBL). WBBLs are important for a variety of nearshore and coastal engineering problems, including sediment transport resulting from turbulence and turbulent bed shear stresses driven by these oscillations. Although much is known about steady flow boundary layers, study of oscillatory boundary layers still garners much research mostly due to the many possibilities for parameterizing the turbulence.

The most numerically simple methods for parameterizing the turbulence are to specify the shape of the profile a priori based on the friction velocity [e.g., Trowbridge and Madsen (1984) and Fredsøe and Deigaard (1992)] whereas the computationally ex-

pensive methods require direct numerical simulations where the model grid is so small that all the necessary scales of motion are calculated rendering a parameterization unnecessary [e.g., Spalart (1988)]. A middle ground approach requires descriptions of the turbulent kinetic energy (TKE) and energy dissipation rate in the model that are used to determine the turbulent eddy viscosity. These turbulence closure schemes (namely  $k$ ,  $k-\varepsilon$ , and  $k-\omega$ ) along with the specified eddy viscosity models will be investigated here by comparison to laboratory data.

## Boundary Layer Theory and Numerical Methods

The time-dependent equation of turbulent motion for an incompressible Newtonian fluid with an ensemble mean defect oscillatory velocity component,  $u_d$ , parallel to the bed is given by

$$\frac{\partial u_d}{\partial t} = \frac{\partial}{\partial z} \left[ (\nu + \nu_t) \frac{\partial u_d}{\partial z} \right] \quad (1)$$

where  $\nu$  = kinematic molecular viscosity of the fluid;  $\nu_t$  = turbulent eddy viscosity in the usual Reynold's averaged sense;  $z$  = vertical coordinate; and  $u_d = u - U$ , represents the difference between the depth-dependent velocity inside the boundary layer,  $u$  and that of the free stream,  $U$ .

A variety of methods have been arrived at to determine or specify  $\nu_t$ . One option is to assume or require the flow be laminar in which case  $\nu_t$  is zero for all time. Many other options exist including a linear form using a time varying friction velocity (Trowbridge and Madsen 1984), a parabolic form (Fredsøe and Deigaard 1992) and the turbulent closure schemes mentioned above. We adopt the one-equation  $k$  and two-equation  $k-\varepsilon$  model

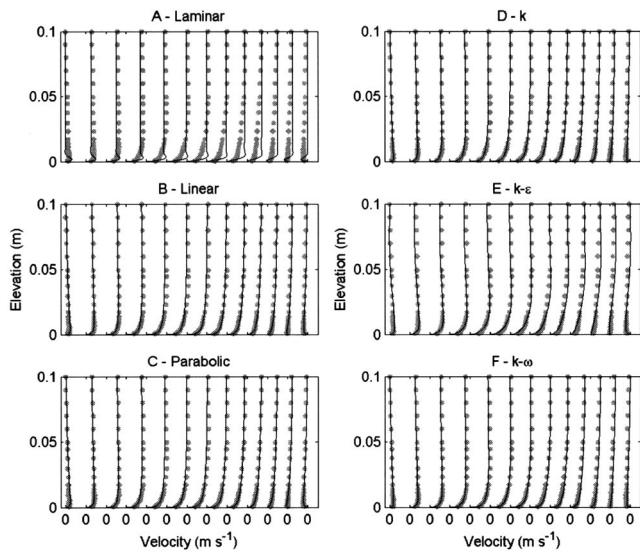
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**Fig. 1.** Rough bed (Test 13), experimental data (dots), model (solid curves): (A) Laminar, (B) linear, (C) parabolic eddy viscosity; and (D)  $k$ , (E)  $k-\epsilon$ , and (F)  $k-\omega$  turbulence closure schemes. Each zero velocity tick mark is offset by  $2 \text{ m s}^{-1}$ .

described in (Wilcox 2000) and the  $k-\omega$  model described in (Wilcox 1988). For brevity here, complete descriptions of the models are not given and the reader is referred to the references above or Rodi (1980), Justesen (1988), or Pope (2000) among many others for further description.

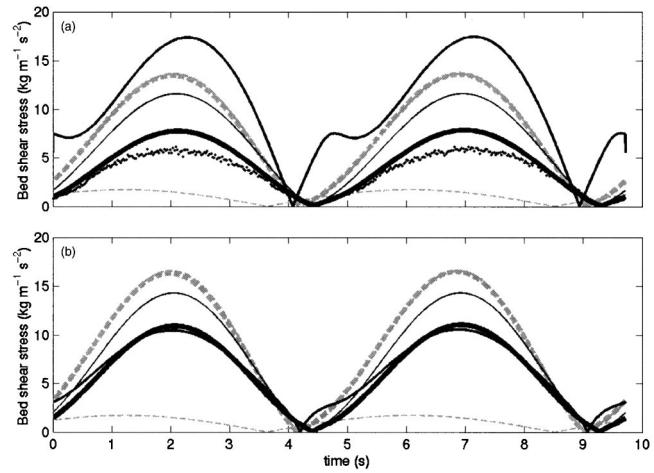
The equations for the boundary layer flow and the various eddy viscosity approaches are solved numerically via an implicit algorithm. A logarithmic vertical grid is used with asymmetrical second-order spatial differences. Full description of the numerical technique, boundary conditions and forcing mechanisms are provided in Puleo et al. (2003).

## Model Results and Model-Data Comparison

The WBBL models will be tested against Test 10 and Test 13 from Jensen et al. (1989). Data was collected at every  $15^\circ$  of wave phase for eighty wave cycles in a U-shaped oscillatory water tunnel using a laser Doppler anemometer (LDA). Test 13 was forced with a sinusoid of  $9.72 \text{ s}$ , a maximum velocity of  $2 \text{ m s}^{-1}$  and an equivalent Nikuradse sand grain roughness of  $K_N=0.84 \text{ mm}$ . Test 10 was forced with the same conditions except the bed was made smooth by the installation of PVC plates. Bed shear stress was also measured for Test 10 using a hot film probe. Because the models require a  $z_0$  value, a smooth bed was approximated by using  $K_N=1 \times 10^{-4} \text{ mm}$  (4 orders of magnitude smaller than the rough bed case and one order of magnitude smaller than clay particle size on the Wentworth scale). Two thousand time steps per wave period and 300 logarithmically spaced grid points were used in our calculations.

### Velocity Profiles

Fig. 1 shows the velocity comparison for every  $15^\circ$  of phase for the first half of the wave for the simulations (solid lines) and the Jensen et al. (1989) Test 13 data (dots). Each successive profile is offset by  $2 \text{ m s}^{-1}$  and is denoted by the zeroes in the plot. Because of the turbulent nature of the flow, the laminar (the term laminar



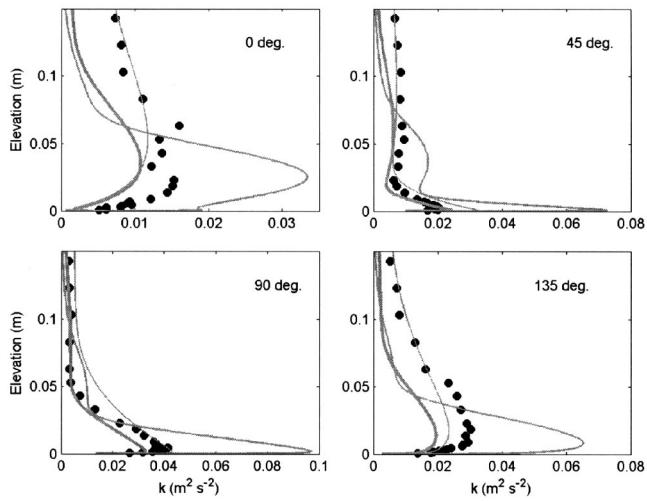
**Fig. 2.** Bed shear stress for (a) Test 10; smooth bed and (b) Test 13; rough bed. Laminar (thin dashed); linear (medium dashed); parabolic (bold dashed);  $k$  (thin solid);  $k-\epsilon$  (medium solid); and  $k-\omega$  (bold solid), Test 10 data (dots).

here refers to the fact that only molecular kinematic viscosity is included in the solution) simulation is not expected to match the observed profile shape and is given for illustrative purposes.

The  $k-\epsilon$  simulation generally has the correct velocity profile shape but fails to predict the magnitude in either the upper portion of the domain or near the bed where it over or under predicts the velocity magnitude at nearly all wave phases. The rest of the simulations are very similar and qualitatively match the data (quantitative comparisons are given in the following). Smooth bed simulation (Test 10) velocity profile comparisons (not shown) are similar to those of Test 13 and suggest the models are qualitatively similar regardless of whether or not they are applied to smooth or rough beds.

### Shear Stress

Bed shear stress estimates for Tests 10 and 13, shown respectively, in Fig. 2(a and b), indicate variation over the period and between the different model formulations. For instance, all stress estimates have similar shapes, but vary in magnitude from a maximum of  $1.7 \text{ kg m}^{-1} \text{ s}^{-2}$  for the laminar simulation to  $17.4 \text{ kg m}^{-1} \text{ s}^{-2}$  for the  $k-\epsilon$  model simulations. The inflection in the  $k-\epsilon$  model bed shear stress estimate is likely attributed to the flow reversal. Bed shear stress measurements for Test 10 are most closely predicted by the  $k-\omega$  model. The predictions match the data for most of the wave cycle except during the periods of highest flow magnitude and capture the correct timing of flow reversals. The  $k$  simulation also captures the correct timing of flow reversals, but over predicts the bed shear stress by up to factor of two. The linear and parabolic eddy viscosity simulations over predict the measured bed shear stress and predict the minima in bed shear stress before they are measured. Finally, the  $k-\epsilon$  model over predicts the measurements by about 500% and is likely the cause for the poor predictive capability of the model for this case. The bed shear stress estimates for Test 13 [Fig. 2(b)] are similar to their smooth bed counterparts except that the  $k-\epsilon$  model estimate overlays the  $k-\omega$  model estimate further suggesting that the  $k-\epsilon$  model has difficulty on the smooth bed simulation.



**Fig. 3.** Turbulent kinetic energy for Test 13 at four phases for  $k$  (thin line),  $k-\varepsilon$  (medium line),  $k-\omega$  (thick line), and data (dots)

### Turbulent Kinetic Energy

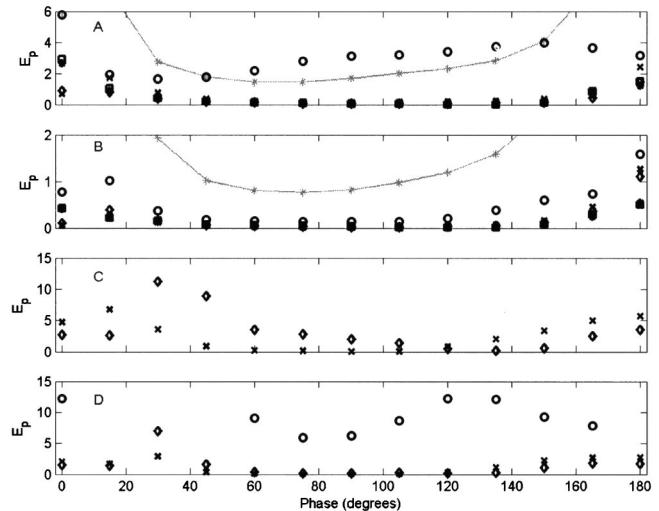
Fig. 3 compares TKE profiles at four phases of the wave cycle for the  $k$ ,  $k-\varepsilon$ , and  $k-\omega$  models to the data for Test 13. TKE was estimated from laboratory measurements as 0.65 times the sum of the mean square velocity fluctuations in the flow-parallel and flow-normal directions (Justesen 1991). It is clear from the simulations that the  $k-\varepsilon$  model has difficulty reproducing the measured TKE near the bed regardless of the phase of the free surface flow. In other words, our implementation of the  $k-\varepsilon$  model shows difficulty in predicting the TKE even away from strong adverse pressure gradients. In contrast, the  $k$  and  $k-\omega$  models more closely match the measurements, even though variations are seen both near the bed and in the upper portion of the boundary layer. Similar results were found for Test 10, but for brevity are not shown here. Overall prediction errors for both tests are described below.

### Eddy Viscosity Models versus Turbulence Closure Schemes

A quantitative measure of the predictive capability of the models can be determined from the normalized phase dependent error (the sum over  $z$  of the estimated error variance normalized by the variance in the data)

$$E_p = \frac{1}{N_z} \sum_z \frac{(\chi_{\text{data}} - \chi_{\text{model}})^2}{\sigma_{\text{data}}^2(\varphi)} \quad (2)$$

where  $\sigma_{\text{data}}^2(\varphi)$  = velocity variance (over the vertical direction) in the measurement data at each phase and  $\chi$  model prediction; in this study either the velocity or TKE. The smooth and rough bed comparisons for velocity show that the linear model was most accurate (although the parabolic,  $k$  and  $k-\omega$  models had similar errors) whereas the laminar model was least accurate (Fig. 4; laminar errors extend beyond the axis range shown). The  $k-\varepsilon$  model had normalized errors that were up to 6 times as large as the  $k$  and  $k-\omega$  models. The errors for velocity become largest between 0 and 20° and between 160 and 180° when the flow is decelerating and passing through flow reversal. Away from flow reversals (adverse pressure gradients), all of the models except the laminar and  $k-\varepsilon$  perform nearly equally for both the smooth bed [Fig. 4(a)] and rough bed [Fig. 4(b)] tests. In general the best



**Fig. 4.** Normalized phase dependent velocity errors: (A) smooth bed Test 10; and (B) rough bed Test 13. Normalized turbulent kinetic energy (TKE) errors for the turbulent closure schemes: (C) smooth bed Test 10 and (D) rough bed Test 13. Laminar (gray asterisk dash); linear (asterisk); parabolic (square);  $k$  (diamond);  $k-\varepsilon$  (circle); and  $k-\omega$  (cross).

performers for velocity were the linear and parabolic eddy viscosity simulations and the  $k-\omega$  model.

Figs. 4(c and d) show the normalized TKE errors but for the  $k$ ,  $k-\varepsilon$ , and  $k-\omega$  models only. Again, it is clear that the  $k$  and  $k-\omega$  models outperform the  $k-\varepsilon$  model especially in the smooth bed simulation [Fig. 4(c)] case where the errors for the  $k-\varepsilon$  model extend well outside the axis range.

The overall predictive capability of the models can be determined from the normalized total error (normalized error variance summed over space and summed over each phase).

This calculation was performed for the velocity only for the linear and  $k-\omega$  model for time steps ranging from 100 to 2,000 per wave cycle to address the predictive error as a function of computation time. It was found that the velocity errors were not statistically different for any of the various time steps. However, the fastest  $k-\omega$  simulation was 1.5 times slower than the slowest linear eddy viscosity simulation and about 40 times slower than the fastest linear eddy viscosity simulation. The same calculations were not carried out for TKE due to difficulty in accurately estimating TKE from the linear eddy viscosity model.

### Conclusions

Six one-dimensional wave bottom boundary layer models have been compared to laboratory data on smooth and rough beds. In general, the eddy viscosity models, where the shape of the vertical profile of eddy viscosity is specified perform just as well or better than one and two equation turbulence closure schemes in predicting the overall velocity profile. In addition, it was found that the model using a linear profile of eddy viscosity yields slightly better results than the more numerically intensive  $k-\omega$  model on a rough bed at about 40 times savings in computer processing requirements. However, the  $k$  and  $k-\omega$  models had a relatively high predictive capability in terms of TKE and the  $k-\omega$  model was the most accurate predictor of bed stress. It must be kept in mind that the findings in this note are for high Reynolds

number ( $>10^6$ ) flows and have not been verified or tested for small Reynolds number flows in this study. In addition, these findings are not easily extrapolated to multidimensional (2D or 3D) models, but suggest that for one-dimensional flat bed (smooth or rough) simulations no increase in predictive capability is obtained in using the  $k$ ,  $k-\varepsilon$ , or  $k-\omega$  turbulence closure schemes over the more simplified linear or parabolic eddy viscosity models with respect to total error variance for velocity profiles. However, if one is interested in accurately predicting TKE or more importantly for sediment transport studies, the bed shear stress, then, of the models tested, the  $k-\omega$  model is recommended.

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## Notation

The following symbols are used in this technical note:

- $E_p$  = normalized phase dependent error between model and data;
- $\varepsilon$  = turbulent dissipation rate;
- $K_N$  = Nikuradse equivalent sand roughness;
- $k$  = turbulent kinetic energy;
- $N_z$  = number of grid points;
- $t$  = time;

- $U$  = free stream fluid velocity;
- $u$  = bed parallel fluid velocity;
- $u_d$  = deflect velocity ( $u-U$ );
- $Z_0$  = zero level for velocity ( $kN/30$ );
- $z$  = distance above horizontal bed;
- $\nu$  = kinematic viscosity;
- $\nu_t$  = turbulent eddy viscosity;
- $\sigma^2$  = velocity variance;
- $\varphi$  = oscillatory flow phase;
- $\chi$  = model prediction for velocity or TKE; and
- $\omega$  = specific dissipation rate ( $\varepsilon/k$ ).

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